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Abstracts from the CECAM Workshop on Computer Simulations of Cellular Automata

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INTRODUCTION

We present the abstracts of the talks given during the workshop on cellular automata and their computer simulations which was held in Orsay.

Cellular automata, although invented over 40 years ago by J. von Neumann, have only recently been applied succesfully to describe processes in nature ranging from cell differentiation in biology to hydrodynamics. Cellular automata are networks which have a discrete variable and a rule on each site. The rule determines the value of the variable at the next time step so that the cellular automaton describes the time development of any configuration of variables. For this reason cellular automata are dynamical systems with many degrees of freedom, and the application of methods from statistical physics to them has been particularly successful in recent years.

In Orsay we organized from September 26 to October 7, 1988, a workshop sponsored by the CECAM (Centre Européen de Calcul Atomique et Moléculaire, Orsay) during which we discussed new developments concerning the applications of cellular automata. More precisely, we emphasised two aspects: how to model phenomena in nature via cellular automata and the use of powerful techniques (like large-scale computer simulations) applied to cellular automata. Since the majority of the participants had a background in statistical physics, there was a common language with regard to numerical methods and the corresponding extrapolation techniques.

The first week of the workshop was devoted mainly to modelizations, two days more specifically to the biological aspects. Given the regulatory system of a genetic process, R. Thomas showed how to cast it into a finite

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automaton. A more generic model for molecular evolution was proposed by L. Peliti. G. Weisbuch explained the essentials of immunology, making clear that the difficult tasks of the immune system can only be performed via feedback loops. Finally, S. Solla gave a short course on the large variety of neural networks that have been invented to describe the various processes going on in the brain.

The group from Florence and P. Tamayo both brought special cellular automata computers, i.e., electronic boards that are plugged into an IBM PC and which act like an independent processor simulating a cellular automaton. The software for the processors resides on the PC and is used to program the rule of the automata and to control the execution on the board. The time evolution of the automaton can be visualized on a color screen with about 50 updates of the whole system per second. F. Bagnoli and A. Fransescato presented the general structure of a cellular automaton computer and gave details on the architecture of the machine built in Florence. P. Tamayo used this machine to show the time development of a very entertaining family of automata which model reactions producing spatial patterns like the Belousov–Zhabotinsky reaction.

Other cellular automata that describe chemical reactions were presented by M. Droz and the particular case of catalyzed oxydation was treated by P. J. Plath.

The solution of hydrodynamic equations is of great technological importance, but their numerical solution often runs into accuracy problems due to roundoff errors. A promissing way out seems to be the lattice gas automata in which particles follow linear trajectories except when they collide. The trick consists in finding the collision rules that yield the correct macroscopic behavior. Y. Pomeau and S. Zaleski presented this exciting development. Another way of simulating fluids, in particular the Rayleigh–Bérnard instability, was proposed by S. Ruffo.

The second week of the workshop was closer to traditional statistical physics. D. Stauffer looked at the final state of all the 65,536 rules on the square lattice with nearest-neighbor inputs. A. Hansen described the transition between the frozen and the chaotic phase as a directed percolation problem.

The spreading of damage was a very popular subject. Two initial configurations that differ only in very few sites (damage) are submitted to the same cellular automaton. Since usual Monte Carlo can be considered as a probabilistic automaton, one can detect in this way phase transitions in the dynamics of, e.g., the Ising model. A. Coniglio showed an exact equivalence between the damage and correlation functions. B. Derrida presented the scenario that emerges for spin glasses. Damage spreading in the Kauffman model is particularly interesting. The sites that never change (stable core)

were calculated by H. F. Flyvbjerg and evidence for the multifractality of the damage probability distribution was given by N. Jan.

The question of how to define the complexity of a dynamical system was discussed by P. Grassberger. He presented many possible definitions, but concluded that none of them is really satisfactory.

Neural networks with or without feedback loops can have a very rich behavior. S. Solla showed how they can learn. A. Zippelius showed the effects that arise if the couplings are asymmetric. K. Kurten stressed more the relation to cellular automata and J. P. Nadal the connection to spin glasses.

A fascinating new automaton that shows self-organized critical behavior was presented by P. Bak. I. Webman also presented a new automaton in one dimension. C. Tsallis gave results on a generalization of the automaton introduced by Domany and Kinzel, and M. Kolb showed evidence that a lattice gas is a good random number generator to the point that one can reproduce the density profile of diffusion.

The workshop ended in a do-it-yourself course on the CAM6, the cellular automaton computer brought by P. Tamayo.

In the following we give the abstracts of the talks.

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Asynchronous Logical Description of Biological Regulatory Circuits

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Biological regulatory systems are based on negative feedback loops which ensure homeostasis and positive feedback loops which permit a choice between two or more attractors; most systems comprise several intertwined loops.

In *logical* descriptions, variables and functions can take only a limited number of values (0, 1, 2,...). We associate with each relevant element of a

system, not only a logical *variable* (a, b, c,...) whose value describes the *level* (e.g., concentration) of the element, but also a logical *function* (A, B, C,...) which describes its *evolution*. For example, in genetics $a = 1$ means that a gene product is present (i.e., above its threshold of efficiency), $a = 0$ that is absent; and $A = 1$ means that the corresponding gene is on (i.e., the rate of synthesis is significant), $A = 0$ that the gene is off. The system is described by logic of the form: $A = f_1(a, b, c...)$, $B = f_2(...)$.

At any time A is a function of the values of a, b, c at *that* very time (no delay); in turn, the value of each variable depends on the earlier value of the corresponding function, with a time delay characteristic for each transition.

This formalism (kinetic logic) can be used for both biological regulatory circuits and asynchronous automata. It has in fact been used not only in various fields of biology, but also in climatology (C. Nicolis) and other disciplines.

Inductive use. Instead of proceeding from model to behavior, one can ask to what extent it is possible to proceed rationally from behavior to models (usually the simplest models which account for the observations).

Generalized method. Now, we use variables and functions with more than two values whenever there is a *qualitative* reason to do so. Moreover, we use *logical parameters* (E. H. Snoussi) which permit us to account for the qualitatively different situations due to the various respective weights of the logical terms. There is a remarkable complementarity between the generalized kinetic logic and differential descriptions.

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A Simple Model of Molecular Evolution

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We consider a population of fixed size M, composed of individuals, the state of each of which is described by a collection of N Ising units σ_i. The

population undergoes a mutation and selection cycle (called generation); first, a smalll number of units have their state changed; second, the fitness H of each individual is evaluated, and the individual is removed with a probability dependent on H:

$$\rho = \{1 + \exp \beta(H - H_0)\}^{-1}$$

The fitness H is a function dependent on the state of the N units of the individual. The gaps of the population are then filled in by making copies of randomly chosen individuals among the surviving ones. The fitness function is chosen to be either of the form of a spin-glass Hamiltonian

$$H = \sum J_{ij}\, \delta_i\, \delta_j$$

where the J_{ij} are independent random variables for each pair of units, or as a fully random function, i.e., one whose value for each of the 2^N possible states is an independent random variable.

The population rapidly assumes a peaked distribution in the space of states. The peak evolves in state space with a diffusion constant independent of the population size. Its evolution may be neutral or adaptive according to the value of the threshold H_0. The sharpness parameter β controls the dispersion of the population (with more peaked populations corresponding to lower values of β). The transition from neutral to adaptive behavior corresponds to a phase transition in a statistical mechanical model corresponding to the Hamiltonian H.

Dynamical Phase Transition in Immune Networks

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The immune system is composed of cells (lymphocytes) and macromolecules (antibodies) that can react with any foreign substance (antigen) in order to destroy it. There exist an immense variety of antibodies in terms of primary sequences and shapes, and only a few species among them react with a given antigen: the chemical specificity of the reaction (the so-called recognition) is based on complementarity of the shapes of the antigen and

the antibody: it is often compared to a lock-and-key selection mechanism. Recognition then triggers the multiplication of those antibodies which are specific to the antigen, which is further destroyed because of their presence.

But the antibody can also play the part of an antigen and elicit a secondary response of other antibodies, which further react against it. Of course, those secondary antibodies themselves elicit a tertiary response and so on. We are then in the presence of a network of interacting chemical and cellular species. We represent it by a network of threshold automata (also called a neural net, or a spin glass at zero temperature). The automata represent populations of cells or antibodies of a given specificity and the intensity of their connections represent their chemical and functional interactions. Due to the scarcity of information about the set of interactions in the immune system and to the difficulties of predicting them from calculations, we used the approach of inverse dynamics.

We definitely do not want the network to have a chaotic behavior, such that any perturbation, such as those introduced by the presentation of only one antigen, would change the state of a finite fraction of the units of the net; such a change would scramble any memory effect due to previous antigen presentations. On the contrary, we want one antigen presentation to elicit changes among a small number of antibody species, in order to control the immune response. The transition between organized and chaotic behavior has been thoroughly studied by Derrida. We applied his results[1] to a network made of threshold automata with a mean threshold of h and with k connections randomly chosen of amplitude $+1$ or -1: such a net remains in the organized regime if the threshold h is larger than $(k \cdot \log k)^{0.5}$.

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Computational Neuroscience

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Some features of biological neurons are captured by model neurons as simple input–output devices that perform a nonlinear transformation on a

linear combination of inputs. Networks of many such devices with a large degree of connectivity exhibit interesting computational abilities, which emerge as collective properties of the system. Feedback networks are particularly useful for recall and associative memory, while layered networks are preferable for categorization, recognition, and diagnosis.

The goal is to understand the emergent properties of the networks through their mechanisms for parallel distributed processing.[1] Experimental neurobiology provides a rich understanding of brain functions at the molecular and cellular level. Such features need to be incorporated into models to describe high-level functions as emerging from the organization and interaction of low-level components.

The utility of simplistic neural models is illustrated by two examples from the mammalian visual system: orientation selectivity in the striate cortex,[2] as described by a feedback network with massive inhibition, and the extraction of shape from shading,[3] as described by a layered neural network.

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Architecture for Cellular Automata Machines

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Cellular automata are the extreme consequence of parallel computation, as they are made of a large number of synchronous processors with fixed connections: each of them can perform only a small set of operations. In the machine designed by Cabibbo, Medici, and Petrarca that our group developed in Florence, a definite architecture is chosen. It is based on a serial scanning of the lattice, a temporary memory from which one can get out the neighborhood of each cell in parallel, and a lookup table to code the rule. The machine is driven by a customary interpreter written in C-language and can be connected to ordinary PC's. Performence is similar

to that of the Toffoli machine.[1] Extensive simulations of the statistical behavior of the "game of life" have been performed on the machine.[2]

The main bottlenecks of this architecture are capacity of the lookup table and impossibility of performing real-time data analysis. From a general point of view an object-oriented and machine-independent software would be preferable. In order to improve performances in this direction, we are designing a more powerful and flexible machine with an open architecture.

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Discrete Models for Reaction-Diffusion Systems and Cellular Automata

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The three-state Greenberg–Hastings model for excitable media produces oscillatory patterns reminiscent of those observed experimentally in the Belousov–Zhabotinsky reaction. A reversible cellular automata based on this model displays a rich variety of space-time patterns. The system evolves in one of three regimes according to the initial conditions chosen: (1) a regular regime characterized by solitons and short recurrence times; (2) a turbulent-like regime with long-range coherent regions; and (3) a random-looking regime without coherence or regularity.[1–3,1]

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Nonequilibrium Phase Transitions and Cellular Automata

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A cellular automata model of surface reaction is discussed. This model describes a simple adsorption-dissociation-desorption process on a catalytic surface. Analytical mean-field-like results as well as "exact" solution obtained by simulation on a special purpose computer are reviewed. This model exhibits two second-order nonequilibrium phase transitions. The stationary critical exponents β characterizing the behavior of the order parameters near the transitions as well as the dynamical critical exponent Δ describing the critical slowing down are computed. The results are interpreted in view of what is known for several other models showing nonequilibrium phase transitions. The problem of the existence and characterization of universality classes for nonequilibrium critical exponents is then discussed.

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A One-Dimensional Cellular Automata As a Model for the Heterogeneously Catalyzed Oxidation of CO

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Experimental results of the catalytic oxidation of CO by Pd single crystals in a realite matrix are reported. There are constraints for which the

stationary state of this system is an oscillatory one. The corresponding time series resembles self-similar patterns. A table is given to show how these patterns can be simulated by a one-dimensional cellular automaton. The level of describing the reaction is carefully discussed to give good reasons for the chosen macroscopic interpretation of the automaton. The sum of cells in the nonactive states of the automaton at time t is just the value which can be compared with the observable of the chemical system. Time series, Poincaré maps and Fourier spectra of the cellular automaton and the chemical system are compared to show the qualitative agreement of both the systems in several properties.[1-4]

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Proposals Toward Simulation of Long-Ranged Cellular Automata

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1. Let us consider a classical system, consisting of n-component unit vectors associated with a d-dimensional (hypercubic) lattice $\{u_u/Ke\, Z^d\}$, and interacting via an isotropic [i.e., $O(u)$-symmetric] translationally invariant pair potential.

$$V_{ik} = f(\mathrm{II}i - k\mathrm{II})\, \Psi(u_i u_k)$$

Some general symmetry properties hold (a) for plane rotators (i.e., $n = 2$) and (b) for any Ψ being an odd function of its argument.

We now specialize the potential to

$$V_{ik} - \varepsilon \mathrm{II}i - k\mathrm{II}^{-d-\rho}(u_i u_k), \qquad \varepsilon > 0, \quad \rho > 0$$

A number of results are known for these models (e.g., existence of

an ordered phase, at low but finite temperature, existence of a Kosterlitz–Thouless transition, critical exponents via RG), whereas very little is known about their antiferromagnetic counterparts, and some open questions remain (e.g., $d=1$, $n=2$, $\sigma=1$).

2. At the simplest level, a one-dimensional cellular automaton with "synchronous updating" is defined by a rule of the form

$$S_k(t+1)=F(S_k(v), S_{k-1}(v))$$

where F is an appropriate Boolean function symmetric with respect to interchange of S_{k-1} and S_{k+1}.

The possible functional forms of F consistent with appropriate constraints ("legal rules") have been classified, and also nondeterministic rules have been studied.

3. I now aim at simulating one-dimensional automata with long-range interactions, in order, so to speak, to bridge the gap between paragraphs 1 and 2. A general form for updating rules can be devised. We have

$$\sigma_k \in \{-1, +1\} \qquad \text{all } k, \quad \text{all } t$$

$$\sigma_k(t+1)=S\{F(\sigma_k, \psi_k)\}$$

$$S(x)=1, \qquad x<0, \quad x=0$$

$$S(x)=-1, \qquad x>0$$

$$\psi_k=\sum g(|i-k|)\,\sigma_i$$

where the series $\sum g(m)$ converges absolutely. There are too many adjustable parameters; we restrict their number by imposing

$$g(m)=m^{-2}$$

$$F\{\sigma_k, \psi_k\}=\alpha_1\sigma_k+\alpha_2\psi_k+\alpha_3\sigma_k\psi_k$$

Additional constraints can be determined by considering some selected configurations.

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Hydrodynamics by Lattice Gas Automata

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Besides their own interest, cellular automata (CA) may be used for modeling physical systems[1] made up of large assemblies of similar interacting entities. The first condition for this is to have something like Hamiltonian dynamics in discrete and fully deterministic systems. The first step in that direction was made by Edward Fredkin, who has defined a fairly general class of reversible CA. This may be extended[2] to a dynamical model of the Ising spin system. Another application of the same concept is to gases and fluids. This was made for a gas on a square lattice[3] then on a hexagonal lattice,[4] which represents fairly well the dynamics of a real fluid on large scales of space and time. This has been extended since to 3d fluids[5] and to various cases with free boundaries[6] that are notoriously difficult to handle by classical methods.

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Fluid Mixtures with Lattice Gas Methods

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Much research has been directed recently to the treatment of fluid mixtures with lattice gas methods. (General references about lattice gas methods can

be found in Pomeau's contibution.) Mixtures of noninteracting species have been studied in ref. 1. A simple diffusive behavior is then obtained. Reacting mixtures have been introduced in which separation in two phases is observed.[2] A more tricky problem is to introduce repulsive interactions between species so that surface tension is also obtained. This was done by the introduction of immiscible lattice gases in ref. 3. A different method that also yields surface tension was proposed in ref. 4. Immiscible lattice gases have given rise to much interest recently because of their potential for the study of problems such as Rayleigh–Taylor instabilities and the invasion of porous rocks by fluids with wettability properties.[5] Spinodal decomposition may also be studied with such methods, either on the Ising model with microcanonical Kawasaki dynamics[6] or with different models recently implemented on the CAM 7 special-purpose machine.[7] These least two models do not conserve momentum, but have an invariant energy and thus ae microcanonical models. The spinodal decomposition in the immiscible lattice gases of ref. 3 was investigated in ref. 8. It is of a different nature because while momentum is conserved, energy is not. All the previously mentioned models have two dimensions of space, but an extension of the immiscible lattice gas to 3 dimensions was described recently.[9]

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A Cellular Automata for the Rayleigh–Bénard Instability

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Recently experiments on Rayleigh–Bénard convection at intermediate Rayleigh number have shown the presence of phenomena reminiscent of phase transitions.[1]

Simulations have been performed in systems of coupled maps which show similar phenomena.[2]

I propose a model for Ciliberto's experiment based on probabilistic cellular automata, which shows a transition from confined to deconfined turbulence.[3]

Some probabilistic behavior of the fluid system is well described, but there remains some degree of determinism which is not all explained.

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Toward a Classification of All Square Lattice Cellular Automata

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All 65,536 automata on the square lattice with nearest-neighbor interactions, ignoring the central spin, were simulated. About 4–8% of them ended in the fixed point "all spins down," the same amount in the fixed

point "all spins up," nearly 3% in periods of two with mixed configurations, and less than 1% in oscillations between all spins up and all spins down, and in fixed points with a mixture of up and down spins. Damage spreading studies are planned. Upon changing the initial concentration of randomly placed up spins from 50 to 90%, about 1000 automata changed their above behavior. A more detailed investigation of the 64 symmetric rules showed four phase transitions, among them two with critical slowing down. Perhaps these transitions vanish logarithmically in the thermodynamic limit.

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Spreading of Damage and Percolation in Deterministic Cellular Automata

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We identify a substructure constructed from the rules on which the damage spreads in a certain class of deterministic cellular automata[1] known as "legal" ones. A legal cellular automaton[2] is defined as having at least one state that maps onto itself. It can be shown that this substructure of the rules must percolate in order for the state that maps onto itself to be

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unstable against damage.[3] We present numerical evidence that in the thermodynamic limit the percolation critical point and the onset of chaos where the cellular automaton becomes susceptible to damage coincide. This evidence is based on extensive computer experiments on several different inhomogeneous cellular automata.

We also present a transformation of general deterministic cellular automata into a form that makes it possible to identify a substructure among the cellular automaton rules that has the same properties as the one identified in the case of legal cellular automata with respect to damage spreading. Also in this case the percolation critical point in the thermodynamic limit coincides with the onset of chaos.[4]

The results show that the damage-spreading phase of deterministic cellular automata may be identified with a percolating phase of the above-mentioned substructure of the cellular automaton rules.

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Damage Spreading in the Ising Model

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We use the concept of damage spreading to characterize in a dynamical way the critical behavior of a ferromagnetic Ising model at the critical temperature. We show that: (a) The damage or Hamming distance between two configurations, respectively with plus and minus boundary conditions, submitted to the heath-bath dynamics using the same random numbers equals the magnetization; (b) the probability to damage at a site at distance r from a fixed damage at the origin is proportional to the pair correlation function, and (c) in the last case the sum of all damaged sites is proportional to the susceptibility. This approach not only provides an efficient way to calculate correlation functions and magnetization, but also

to produce a geometrical picture of spin fluctuations. These are visualized as clusters of damaged sites which have a fractal dimension $d-\beta/\upsilon$ at T_c, where d is the Euclidean dimensions and β and υ are Ising exponents. Finally, we also express the size distribution of the clusters of damaged sites in a scaling form which contains the static Ising critical exponents.

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Dynamical Phase Transitions in Networks of Random Automata[1]

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By comparing the evolution of two spin configurations for networks of random automata, one can observe a dynamical phase transition from a frozen to a chaotic phase.[2] The chaotic phase is characterized by the fact that the distance between two spin configurations does not vanish in the long-time limit and by a continuous part in the distribution $P(m)$ of local magnetizations.[3] For the Kauffman model, which is a mean-field model, the distance can be calculated analytically[2] and one can write an exact integral equation[3] for $P(m)$. In finite dimension, no analytical work is available, but numerical calculations[4] of the distance and of $P(m)$ give clear evidence for a transition from a frozen to a chaotic phase. When one introduces a temperature, one can compare two configurations subjected to the same thermal noise.[5] One finds a transition at a finite temperature T_c. Above T_c, the distance vanishes, whereas below T_c, it has a finite limit independent of the initial distance.

[1] The work of which this is an abstract was presented elsewhere.[1]

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Some Exact Results for Kauffman's Model

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A brief historical introduction to Kauffman's model is given. It is explained how Kauffman tried to model the complicated genetic regulatory system of living organisms by a random network of deterministic automata, and how he compared results of his model with observations for living organisms.[1,2]

Kauffman's model is a simple dynamical system with quenched disorder. The basins of attraction of its limit cycles form a multivalley structure which is compared with that of spin glasses, the Sherrington–Kirkpatrick model to be specific. A striking similiarity is observed.[3]

The stable core of a network of automata is defined as the set of variables which after a transient time acquire a constant value that does not depend on initial data. It is shown how the size of the stable core may be computed analytically and used as an order parameter for the transition between the frozen and chaotic dynamics possessed by the model.[4]

The equation for the stable core is generalized to give an equation for the spectrum of local magnetizations $P(m)$. Unexplained properties of $P(m)$ are pointed out.[5]

In its totally connected version Kauffman's automaton becomes a random map in configuration space. Due to the absence of correlations in time in this case, a number of properties can be calculated analytically, among them $g(W)$ (=the probability that a randomly chosen initial configuration belongs to a basin of attraction of relative size W) and $\pi_p(Y_p)$ (Y_p being the probability that P randomly chosen configurations belong to the same basin of attraction. Y_p is sample dependent with distribution

$\pi_p(Y_p)$. Singularities and associated exponents in $\pi_p(Y_p)$ may be found analytically, and comparison with the equivalent distributions for the Sherrington–Kirkpatrick model and for randomly broken objects show that the presence and location of these singularities are universal.[6,7]

Finally, it is shown that also for connectivity $K=1$ Kauffman's model is exactly solvable, and it is conjectured that this solution of the model may be used to solve it everywhere in its frozen phase, and may be even on the phase boundary.[8]

For a recent review of the subjects mentioned here see ref. 9.

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Multifractality in the Kauffman Model

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We check how an initial disturbance, called the damage, spreads through a square lattice of Kauffman cellular automata at the critical point, $P_c = 0.29$. We determine the moments of the probability that a site has been damaged n times, and check for multifractality in the fractal dimensions of these moments of the damage probability vs. (a) the length L of the lattice and (b) time.

The ensemble consists of only these clusters that touch the edge of the lattice. Specifically, we find no evidence for multifractal behavior when the moments of the probabilities are evaluated with lattice size L, but multifractality occurs when the moments are monitored as a function of time.

This feature reflects the property that the frequency of the first passage time has a pronounced peak and an extremely long tail quite similar to that observed for the voltage distribution and random resistor network. This broad distribution and the absence of a unique characteristic time give rise to the multifractal behavior.

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Self-Generated Complexity Studies by Means of Cellular Automata

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I review proposed definitions of "complexity" which might be applicable to situations where this complexity seems to arise spontaneously.

I first point out some properties which such a definition should obey. Among others, it should be observable, i.e., measurable in experimental situations. It should locate complexity somewhere between order and randomness. It should classify objects as complex if they involve hierarchies (in particular, "tangled," i.e., broken, ones), if they involve different-level concepts, or if they involve strong correlations or constraints between their parts. It is proposed that complexity of an object is a measure of the difficulty of some meaningful and important task done on the object. Examples are at times needed for reconstructing the object from its fastest defining algorithm ("time complexity") or from its shortest algorithm ("logical depth"), difficulties of making optimal forecasts of time sequences ("forecasting complexity"), and the amount of storage needed to store its shortest algorithm ("algorithmic complexity").

It is argued that no unique definition of complexity is to be expected, and that furthermore the desired distinction between randomness and com-

plexity makes the latter a subjective quantity. It depends not only on the object, but also on what the observer considers as "meaningful."

These considerations are illustrated with simple one-dimensional cellular automata. It is a particular shown that some of these automata lead to unexpectedly large complexities.

A printed version of this talk will appear in *Helvetica Physica Acta* (Proceedings of Gwatt workshop, October 1988).

Learning and Generalization in Layered Neural Networks

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Layered neural networks are of interest due to their ability to implement input–output mappings. The configuration space of all possible couplings for a fixed network architecture defines a probability distribution over the space of realizable mappings. The entropy of such distribution is an intrinsic property of the network architecture and a measure of its diversity.

Supervised learning leads to an exclusion of the regions of configuration space corresponding to the realization of mappings incompatible with the training set. This process results in a narrowing of the probability distribution over the space of input–output mappings, and a decrease of the associated entropy. It is this residual entropy that limits the generalization ability of the resulting network. Learning can thus be described as an organizational process leading to entropy decrease, and its efficiency measured by the entropy decrease per example in the training set.

The choice of a network architecture to implement a specific mapping is guided by two criteria: *realizability*, in that the architecture must be intrinsically capable of realizing the desired mapping; and *specificity*, in that among all networks with such capability, those with lower intrinsic entropy are to be preferred to facilitate the learning process.

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Spin-Glass Features of Asymmetric Neural Networks

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There are various ways characterize the low-temperature phase of a symmetric Gaussian spin glass with long-range interactions: nonzero local magnetic moments, a time-persistent part of the spin autocorrelation, a divergent spin-glass susceptibility, anomalous slow relaxation, an exponential number of metastable states, the distribution of weights of the many equilibrium states, the distance between configurations, etc. I discuss which of these features persist in (a) models with asymmetric, short-range Gaussian correlations of the couplings (but still infinite-range d interactions) and (b) neural networks, which are asymmetrically diluted. I furthermore present an example of a network with asymmetric couplings, which does show a spin-glass phase, characterized by time-persistent correlations.[1]

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Phase Transitions and Self-Organization Automata

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For diluted networks of random McCulloch–Pitts threshold automata a dynamical phase transition from a chaotic to an ordered phase can be observed by comparing the time evolution of two initial configurations in the thermodynamic limit. The chaotic phase is characterized by an exponential increase of the mean cycle length with the total number of cells, whereas the ordered phase shows a power-law increase. At the critical point the behavior is intermediate. The model is also shown to share behavior and formal similarities with the Kauffman model.

I further study systems with geometrically correlated couplings, such as nearest-neighbor and distance-dependent interactions, where to date no theoretical predictions exist. Computer simulations show that threshold automata residing on regular two-dimensional lattices exhibit by far more ordered behavior than their infinite-range counterparts. Automata with distance-dependent couplings show a phase transition when the interaction range decreases to a size comparable to a few nearest neighbors.

Hebbian-like self-organization routines applied to any of these networks tend to reduce the appearance of chaotic modes substantially.

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Dynamics of Neural Networks: Stability, Symmetry, and Self-Coupling

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We report recent results obtained on the dynamics of networks of formal neurons.[1-3]

For a network of the perceptron type (one input layer and one output layer) we isolate two important sets of parameters which render the network fault tolerant (existence of large basins of attraction) in both heterossociative and autoassociative systems and study the size of the basins of attraction (the maximal allowable noise level still ensuring recognition) for sets of random patterns. These parameters are the "stabilities" of the stores patterns and the set of diagonal couplings. As a result, optimal associativity is obtained by computing the couplings with the "Minoverlap algorithm" introduced recently.

For the full dynamics on a homogeneously connected network with no diagonal couplings, we show that a good estimate of the size of the basins of attraction is obtained by taking into account only the stabilities and the degree of asymmetry of the connections. The effect of the diagonal couplings is also analyzed.

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Self-Organized Criticality

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We have studied a cellular automaton describing a threshold diffusion process. The automaton evolves into a critical point with no length or time scales. We suggest that systems in nature with self-similar fractal structure and power-law $1/f$ temporal correlations can be visualized as dynamical systems at the self-organized critical point. For instance, the power-law distribution of energy released at earthquakes, known as the Gutenberg–Richter law, may be a manifestation of the earth's crust being in a critical state.

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Critical Behavior of Systems with Random Pinning

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The dynamics of a network of particles connected by harmonic springs subject to random local pinning forces and driven by a uniform external field E is studied analytically. The pinning forces act similarly to a local friction: a particle moves only if the total force acting on it is greater than a local pinning strength. These local pinning strengths are assigned at random to the particles (quenched randomness). The theory predicts a critical behavior at a threshold field E_c. Below threshold, large domains of mobile particles nucleate and grow, and the system relaxes to a static state long after the field is switched on. In the static state the size of the largest domains diverges at criticality, and so does the total polarization (the sum

of the displacements of the system particles). During the slow relaxation to the final state, two major processes go on: (a) The formation and growth of mobile domains to their final size. (b) Further relaxation of the particles in stabilized domains. The relaxation of the polarization to the asymptotic value follows a power law growth succeeded by a stretched exponential time dependence. The dynamics shows a critical slowing down characterized by a time scale which diverges as E approaches E_c, while at E_c the polarization grows indefinitely following a power law in time with an exponent smaller than unity. The static and dynamical exponents related to the critical behavior of the system are obtained analytically. Above four dimensions the polarization does not diverge, but the critical slowing down of the dynamics persists. The core of the theory is a derivation of the asymptotic distribution of domains sizes $p(L)$ based on the set of conditions for the arrest of domain growth at a size L. Graphics obtained by numerical simulations of two-dimensional networks which illustrate the nature of the dynamics are presented. The theory is relevant to the very slow relaxation and to the giant polarization observed in recent experiments on charge density waves below the critical external field. More generally, this model may serve as a physical paradigm for the general concept of hierarchically constrained dynamics in glassy systems.

Criticality of a Stochastic One-Dimensional Cellular Automaton

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We consider a generalized version of the stochastic one-dimensional cellular automata studied by Kinzel.[1] It recovers Wolfram-like deterministic cellular automata as particular cases, and might present phase transitions between an absorbing (frozen) phase and a chaotic one. Through computational simulations we study te critical surface in the full parameter space as well as the universality classes (characterized by the behavior of the "order parameter" in the neighborhood of the critical surface).

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Cellular Automata Approach to Inhomogeneous Diffusion and Gradient Percolation

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A deterministic approach is presented for lattice diffusion in two dimensions.[1] This method is implemented on a cellular automaton special purpose computer (CAM-6) in order to study the properties of the interface of particles diffusing from a source to a sink. Care has been taken to fully exploit the parallel architecture of the CA machine.[2] To show that geometrical properties are not affected by the deterministic implementation, fractal properties of the diffusion front of this out-of-equilibrium process are compared with results from inhomogeneous percolation theory.[3] The observed agreement indicates that diffusion fronts and gradient percolation coincide asymptotically and that the CA method is a viable alternative to standard simulations for this class of problems.

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